MECHANICAL MOBILITY TECHNIQUE

Pertti Hynnä

Customer: Tekes / VÄRE technology programme

VTT TECHNICAL RESEARCH CENTRE OF FINLAND

VTT INDUSTRIAL SYSTEMS

A	Work report	
B	Public research report	X
	Research report, confidential to	

Title

MECHANICAL MOBILITY TECHNIQUE

Customer or financing body and order date/No. Tekes VÄRE technology programme	Research report No. BVAL37-021228
Project LIIKKUTEHO (Power methods in control of sound and vibration)	Project No. V9SU00184
Author(s) Pertti Hynnä	No. of pages/appendices 31 /

Keywords

Mechanical mobility, mobility techniques, structure-borne sound, vibrations, transmission

Summary

The structure-borne sound transmission of a built-up mechanical structure can be analyzed using e.g., Finite Element Methods, theoretical and experimental modal analysis methods and Statistical Energy Analysis. The method is selected depending on application and situation. The analyzis is simplified if parts of the system can be analyzed separately. This technique is called substructuring. In practice the substructuring technique should allow the use of measured input data of assembled parts. So-called impedance or mobility techniques allow substracturing and use of measured input data.

This short review presents basic concepts of mobility technique. First mechanical impedance and mobility of elements is discussed. Also impedance and mobility parameters are included. Thereafter transfer matrices, which describe the force and velocity relationships of connected beam elements, is discussed. Multiport methods or transfer function methods are suited for the analysis of built-up mechanical structures coupled in junctions with small dimensions. Emphasis is given to applications in the ship context.

Date Espoo, 5 November 2002

Pekka Koskinen Deputy Research Manager	Pertti Hynnä Research Scientist	Checked

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Foreword

VÄRE - Control of Vibration and Sound - Technology Programme 1999-2002 is a national technology programme launched by the National Technology Agency (Tekes). It raises the readiness of companies to meet the stricter demands set by market for the vibration and sound properties of products.

Mechanical mobility is a useful tool in the analysis of the generation and the transmission of vibrations in structures. It is needed for instance when estimating structure-borne sound power transmission in built-up mechanical structures. Also the characterization of machines as a source of structure-borne sound needs the data of the source mobilities. Other applications include the minimization of vibrational power transmission to seating structures.

This short review belongs to the VÄRE subproject "Vibration and noise control of transport equipment and mobile work machines" (LiikkuVÄRE). The main emphasis is given to general aspects of mobility technique and to the applications in the vibrational power transmission analysis. The other important aspect is how it could be used to minimise the sound transmission from machines via foundation to the receiving structure.

This project is financed by Tekes (that is the main financing organization for applied and industrial research and development in Finland), VTT Manufacturing Technology (nowadays VTT Industrial Systems) and Finnish companies: diesel engine manufacturer Wärtsilä NSD Finland Oy (nowadays Wärtsilä Finland Oy), ship yards Kvaerner Masa-Yards Inc., and Aker Finnyards Oy.

Licentiate in Technology Tapio Hulkkonen from Kvaerner Helsinki yard leaded the supervising group from the beginning to 10th of November 2000. Thereafter Master of Science in Technology Peter Sundström from Wärtsilä Finland Oy took the leadership. The other members of the supervising group have been: Masters of Science in Technology Jouko Virtanen, Berndt Lönnberg and Engineer Juhani Siren from Kvaerner; Masters of Science in Technology Jukka Vasama, Kari Kyyrö and Jari Lausmaa from Aker Finnyards Oy; and Master of Science in Technology Petri Aaltonen from Wärtsilä Finland Oy; project manager of "Vibration and noise control of transport equipment and mobile work machines" (LiikkuVÄRE) Master of Science in Technology Teijo Salmi and project director Doctor of Technology Matti K. Hakala from VTT Industrial Systems.

This study was financially supported by Tekes and Finnish companies, which made this project possible. This project was also supported financially by the Tekes VÄRE-project EMISSIO (Control of noise emission in machinery) through its subproject EMIPOWER (Diesel motor as a source of structure-borne sound). The project manager Master of Science in Technology Kari P. Saarinen made co-operation with this project possible, which is greatly appreciated.

The author wants to express his warm thanks to the supervising group for the support and encouragement during the work.

Espoo, 5 November 2002
Pertti Hynnä

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1 Introduction

Low noise design ([1], [2], and [3]) and design of quiet structures [4] are getting more emphasis at the design stage and it has an important impact on the marketability and competitiveness of many products. At the design stage it is often asked to predict the resulted sound levels of a machine or component. To do this, one needs to analyze all the excitation sources and their interactions with the structures and transmission paths. For air-borne sound well-established design and measurement methods are nowadays available. Instead for structure-borne sound the methods for characterization of machines or components as sources of structure-borne sound are still in phase of intensive research and development (see e.g., references [8] - [18], [29] - [37], [38] -[40], and [41]).

Analysis methods of structure-borne sound transmission must be able to handle with different parts of the built-up structure and with the whole structure as well. Substructuring technique allows the properties of the assembled structure to be calculated using the properties of its parts. In the modelling using substructuring one important requirement is to be able to utilize also measured input data because the properties of mechanical parts vary and sometimes the only way is to use measured input data. The so-called impedance or mobility technique allows both substructuring and use of measured input data [5]. The roles of experiments with computational experimentation are thoroughly discussed by Frank Fahy in [6]. This report is closely related to the state-of-the-art -review "Vibrational power methods in control of sound and vibration" made in this same research program [7]. However, for easiness of reading some basic concepts and definitions are repeated here.

2 Goals

The topic of this short literature review is the applications of the mechanical mobility technique. The main emphasis is given to general mobility concepts and how this technique could be used as a means to study the generation and transmission of vibration in mechanical built-up structures. Also the analysis of vibrations in mechanical built-up structures using mobility technique is considered. It is well known that the transmitted power from source to the receiving structure is dependent on the mobilities of the source and the receiver [41]. That is why the applications of this technique are also sought in the context of machine seating design (see e.g., [5]).

3 Mechanical mobility of lumped elements

Real structures or physical systems can be modelled using idealized mechanical elements with lumped constants. Such elements are mechanical resistance (or damping), spring, and mass. For these elements the mobilities are [28]:

resistance:
$$Y_c = \frac{1}{c}, \tag{1}$$

where c is mechanical resistance with SI unit $[(\mathrm{N}\cdot\mathrm{s})/\mathrm{m}]$;

spring:
$$Y_k = \frac{i\omega}{k}, \tag{2}$$

where k is spring stiffness with SI unit [N/m]; and

mass:
$$Y_m = \frac{1}{i\omega m} = \frac{-i}{\omega m}, \tag{3}$$

where m is element mass with SI unit [kg]. The mobility magnitude as a function of frequency of these idealized mechanical elements is presented in Figure 1.

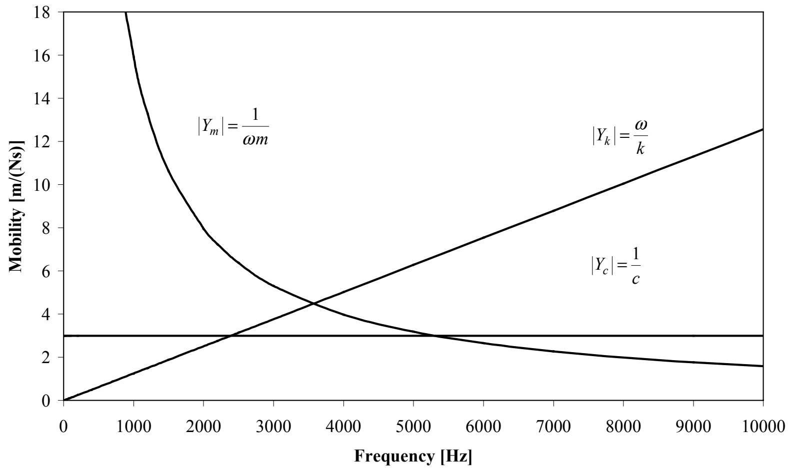


Figure 1. The mobility magnitudes of an ideal mechanical resistance $|Y_c|$, spring $|Y_k|$ and mass $|Y_m|$ as a function of frequency.

3.1 Impedance and mobility of connected elements

3.1.1 Impedance and mobility of parallel connected elements

The properties of mechanical systems can be analyzed using mechanical impedance or mobility concepts. In the analysis impedance and mobility are determined at the points of force excitation or paths for transmitting forces, or at points of common velocities [28]. In Figure 2 the exciting force *F* causes the common velocity at the connection point A, which is common for the parallel connected spring and resistance.

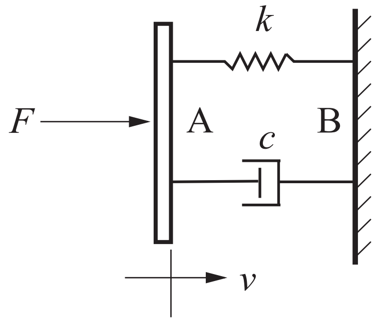


Figure 2. Schematic presentation of a mechanical system consisting of a spring (spring stiffness *k*) and a mechanical resistance (resistance *c*) connected parallel at points A and B. The exciting force *F* gives a common velocity *v* for both elements at point A referred to the stiff point B.

The force required giving the resistance the velocity *v* is obtained from [28]

$$F_c = \frac{v}{Y_c} = \frac{v}{1/c}.$$

The force required giving this same velocity for the spring is obtained from [28]

$$F_k = \frac{v}{Y_k} = \frac{v}{i\omega/k}.$$

The total force *F* is the sum of parallel forces

$$F = F_c + F_k = v\left(\frac{1}{1/c} + \frac{1}{i\omega/k}\right) = \frac{v}{Y_p}.$$

From this it is seen that the inverse of the total mobility Y_p of the parallel-connected elements is

$$\frac{1}{Y_p} = \sum_{i=1}^{n} \frac{1}{Y_i}, \tag{4}$$

where *n* is the number of parallel-connected elements. The total impedance Z_p of parallel-connected elements can be obtained from [28]

$$Z_p = \sum_{i=1}^{n} Z_i, \tag{5}$$

where *n* is the number of parallel-connected elements with mechanical impedances Z_i.

3.1.2 Impedance and mobility of series-connected elements

Another basic way of connecting elements is series connection, which is presented in Figure 3.

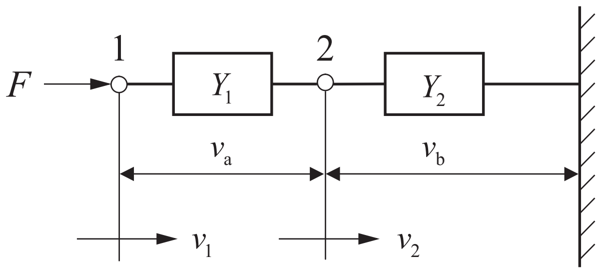


Figure 3. A system consisting of two series-connected mechanical elements with mobilities Y_1 and Y_2. The exciting force F gives relative velocities v_a and v_b between the end connections of each element. The velocities of the connection points 1 and 2 relative to the stationary reference are v_1 and v_2.

The velocities of the connection points 1 and 2 are:

$$v_2 = v_b \text{ and } v_1 = v_2 + (v_1 - v_2) = v_b + v_a .$$

The total mobility at the point number 1 is $Y = v_1 / F$, and the same force F is acting on both elements. The relative velocities expressed with mobilities are:

$$v_a = FY_1 \text{ and } v_b = FY_2 .$$

So the total mobility is

$$Y = \frac{v_1}{F} = \frac{v_a + v_b}{F} = \frac{FY_1 + FY_2}{F} = Y_1 + Y_2 .$$

The total mobility Y_s of n series connected elements is thus

$$Y_s = \sum_{i=1}^{n} Y_i , \tag{6}$$

where Y_i is the mobility of element i.

The total impedance Z_s of ideal series connected elements is obtained from

$$\frac{1}{Z_s} = \sum_{i=1}^{n} \frac{1}{Z_i} , \tag{7}$$

where n is the number of series-connected elements with mechanical impedances Z_i.

4 Impedance and mobility concepts

4.1 Generalized mechanical mobility

Consider a linear and time invariant system, which is excited by a force field $\hat{f} \cdot \exp(i\omega t)$ expressed as complex force amplitude $\hat{f}$ times a harmonically varying function of time $\exp(i\omega t)$. Owing to the system linearity the corresponding velocity field is also harmonic, $\hat{v} \cdot \exp(i\omega t)$, and the ratio $\hat{v} / \hat{f}$ is independent of the amplitude of the exciting force [5]. When the excitation is harmonic at angular frequency ω, the generalized mechanical mobility can be defined as the ratio between the complex amplitudes of the velocity field and the force field as [5]

$$y(\omega) = \frac{\hat{v}(\omega)}{\hat{f}(\omega)} . \tag{8}$$

4.2 Mechanical mobility

ISO 7626-1 [19] defines mechanical mobility Y_{ij} as: "The frequency-response function formed by the ratio of the velocity-response phasor at point *i* to the excitation force phasor at point *j*, with all other measurement points on the structure allowed to respond freely without any constraints other than those constraints which represent the normal support of the structure in its intended application." The definition is given mathematically in Eq. (9)

$$Y_{ij} = v_i \big/ F_j , \tag{9}$$

where v_i is the velocity-response phasor at point *i* and F_j the force phasor at point j.

The velocity response can be either translational or rotational, and the excitation can be either a rectilinear force or a moment. Frequency response function is defined as the frequency dependent ratio of the motion-response phasor to the phasor of the excitation force [19].

Mechanical mobility is sometimes called mechanical admittance. Likewise the complex mobility *Y* can be written as

$$Y = G + iB , \tag{10}$$

where the real part *G* is called the conductance and the imaginary part *B* the susceptance. The SI unit of mechanical mobility is $[\mathrm{m/(N \cdot s)}]$.

4.3 Driving-point and transfer mobility

Direct (mechanical) mobility or driving-point (mechanical) mobility Y_{ii} is the complex ratio of velocity and force taken at the same point in a mechanical system during simple harmonic motion [20]. Here point means both a location and a direction. Sometimes the term co-ordinate is used instead of point. Transfer mechanical mobility Y_{ij} is the complex ratio of the velocity v_i measured at the point i in the mechanical system to the force excitation F_j at the point j in the same system during simple harmonic motion [20].

4.4 Impedance

Impedance is defined as the complex ratio of a harmonic excitation of a linear system to its response during simple harmonic vibration. Both the excitation and the response are complex and their magnitudes increase linearly with time at the same rate [20].

4.5 Mechanical impedance

The mechanical impedance is defined in a mechanical system as the complex ratio of force to velocity as [21], [22]

$$Z = \frac{\hat{F}}{\hat{v}}, \tag{11}$$

where $\hat{F}$ is the phasor of the exciting force, and $\hat{v}$ is the phasor of the velocity as a response at the excitation region. The mechanical impedance is generally a complex function of frequency, because the force and resulting velocity vary with frequency. This general definition is not unique, because the excitation region can be a finite area, and the velocity can vary within this area.

4.6 Moment impedance

In practical situations the force impedance does not cover all the possibilities. Especially this is true in the case of flexural vibrations when moments and angular velocities are of equal importance as forces and translational velocities. The moment impedance W is defined with the exciting moment M and the resulting angular velocity ω as [21]

$$W = \frac{M}{\omega}. \tag{12}$$

The force and moment impedances are not enough to describe completely the response for a point excitation; in general a coupling term is needed [21]. So one must consider carefully if there is only a force or moment excitation and if any coupling terms are needed.

4.7 Driving-point and transfer impedance

Driving-point impedance of a linear mechanical system undergoing sinusoidal vibration is defined as the complex ratio of the exciting force to the velocity response when both are taken at the same point [20]. Here point means both a location and a direction.

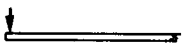
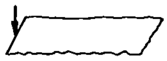
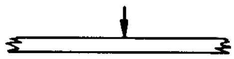
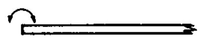
When measuring the point impedance of beam or plate structures, the diameter of the contact area should be less than one-tenth of a flexural wavelength, and not much less than the thickness of the structure under measurement [21].

Transfer impedance of a linear mechanical system undergoing sinusoidal vibration is defined as the complex ratio of the exciting force to the velocity response at another point [20]. Here point means both a location and a direction.

4.7.1 Driving-point impedances of infinite plates and beams

In Table 1 is given some impedance formulas of practical interest in noise control work. In many practical problems these impedance formulas can be applied for finite built-up structures when the frequency is high enough as often is when consideration is given to structure-borne sound.

Table 1. Driving-point impedances of infinite beams and plates [21].

Beam, longitudinal		$Z = S\sqrt{E\varrho} = \varrho S c_L = \omega \varrho S \frac{\lambda_L}{2\pi}$
Thin beam [1]		$Z = \frac{1}{2}\varrho S c_B(1+j) \approx 0.67\,\varrho S\sqrt{c_L h f}\,(1+j) \approx \omega\varrho S\frac{\lambda_B(1+j)}{4\pi}$
		$Z = 2\varrho S c_B(1+j) \approx 2.67\varrho S\sqrt{c_L h f}\,(1+j) \approx \omega\varrho S\frac{\lambda_B(1+j)}{\pi}$
Thin Isotropic plate		$Z = 8\sqrt{B'\varrho h} \approx 2.3 c_L \varrho h^2 \approx \omega\varrho h\frac{\lambda_B^2}{5}$
		$Z = 3.5\sqrt{B'\varrho h} \approx c_L \varrho h^2 \approx \omega\varrho h\frac{\lambda_B^2}{11.5}$
Thick Beam		see Eq. (78b) for $\frac{1}{Z}$
Thick Plate		see Eq. (78a) for $\frac{1}{Z}$
Thin beam [1]		$W = \frac{1}{2}\varrho S c_B\frac{(1-j)}{k_B^2} \approx 0.03\varrho S\sqrt{c_L^3 h^3}\frac{(1-j)}{\sqrt{f}}$
		$W = 2\varrho S c_B\frac{(1-j)}{k_B^2} \approx 0.12\varrho S\sqrt{c_L^3 h^3}\frac{(1-j)}{\sqrt{f}}$
Thin Isotropic plate [1]		$\frac{1}{W} = \left(1 - \frac{4j}{\pi}\ln 0.9ka\right)\frac{\omega}{16B'} \approx \left(1 - \frac{4j}{\pi}\ln 0.9ka\right)\frac{4.8f}{c_L^2\varrho h^3}$
		$\frac{1}{W} = (1 - 1.46j\ln 0.9ka)\frac{\omega}{5.3B'}$

[1] Flexure, or predominantly flexure.

4.8 Driving-point mobility

Mechanical driving-point mobility is defined as the frequency response function formed by the ratio of the velocity response phasor at point i to the excitation force phasor at the same point i. The velocity response can be either translational (velocity v) or rotational (angular velocity $\omega = \dot{\theta}$, where θ is angular displacement), and the excitation force can be either a rectilinear force F or torque T.

4.8.1 Driving-point mobility of infinite plates and beams

The driving-point mobilities given in Table 2 apply to infinite structures in which no resonances can occur. In real finite structures there are always reflections from discontinuities e.g. from junctions and these usually give a resonant response. The damping in the structure controls the magnitude of the vibration amplitudes in the resonance. Usually the largest amplitude will occur at the first resonance frequency. So the largest error in the response will occur at the resonance frequency if the finite structure is replaced by an equivalent infinite structure. The driving-point mobility of a finite structure (for $e^{-i\omega t}$ notation) can be written as [23]

$$\frac{v}{F} = -i\omega \sum_N \frac{[\psi_n]^2}{\omega_n^2(1-i\eta)-\omega^2}, \tag{13}$$

where ω_n is the real resonance angular frequency, η the hysteretic loss factor, n the mode number, and ψ_n is the amplitude of the mode shape. If the damping is small then the effect of the off-resonance terms is small on the amplitude. For a finite structure approximate driving-point mobility can be written as

$$\frac{v}{F} = \frac{[\psi_n]^2}{\omega_n \eta}, \tag{14}$$

if the spacing between resonances is large [23].

If the largest peak mobility values are used when a finite structure is modelled using mobilities of an infinite structure, then the worst case and the largest errors are obtained [23]. The ratio of the point mobility of a finite structure to the point mobility of an infinite structure is also represented in Table 2. From this table it is seen that in practical cases a good approximate mobility is obtained if the properties of an infinite structure is used when estimating the properties of a finite structure.

Table 2. Properties of infinite system (notes: *, torque applied about axis parallel to l_2; time dependence of form $e^{i\omega t}$ assumed) [23].

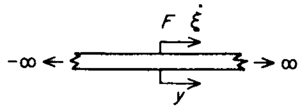
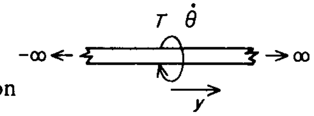
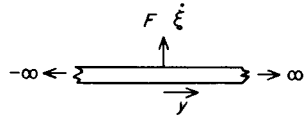
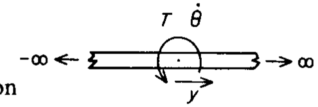
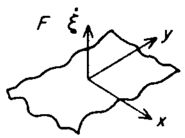
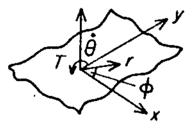
System		Driving point mobility	Power flow into system (P_s) force or torque source	Power flow into system; velocity or angular velocity source
Beam longitudinal wave motion; force excitation		$\frac{\dot{\xi}}{F} = \frac{1}{2A\sqrt{E\rho}}$	$P_s = \frac{\lvert F\rvert^2}{4A\sqrt{E\rho}}$	$P_s = 4\lvert\dot{\xi}\rvert^2 A\sqrt{E\rho}$
Beam torsional wave motion; torque excitation		$\frac{\dot{\theta}}{T} = \frac{1}{2\sqrt{GQJ}}$	$P_s = \frac{\lvert T\rvert^2}{4\sqrt{GQJ}}$	$P_s = 4\lvert\dot{\theta}\rvert^2\sqrt{GQJ}$
Beam flexural wave motion; force excitation		$\frac{\dot{\xi}}{F} = \frac{(1-i)}{4A\rho\sqrt{\omega}}\left(\frac{A\rho}{EI}\right)^{1/4}$	$P_s = \frac{\lvert F\rvert^2}{8A\rho\sqrt{\omega}}\left(\frac{A\rho}{EI}\right)^{1/4}$	$P_s = \lvert\dot{\xi}\rvert^2 A\rho\sqrt{\omega}\left(\frac{EI}{\rho A}\right)^{1/4}$
Beam flexural wave motion; torque excitation		$\frac{\dot{\theta}}{T} = \frac{(1+i)\sqrt{\omega}}{4EI}\left(\frac{EI}{\rho A}\right)^{1/4}$	$P_s = \frac{\lvert T\rvert^2\sqrt{\omega}}{8EI}\left(\frac{EI}{\rho A}\right)^{1/4}$	$P_s = \frac{\lvert\dot{\theta}\rvert EI}{\sqrt{\omega}}\left(\frac{\rho A}{EI}\right)^{1/4}$
Plate flexural wave motion; force excitation		$\frac{\dot{\xi}}{F} = \frac{1}{8\sqrt{B_p\rho h}}$	$P_s = \frac{\lvert F\rvert^2}{16\sqrt{B_p\rho h}}$	$P_s = 4\lvert\dot{\xi}\rvert^2\sqrt{B_p\rho h}$
Plate flexural wave motion; torque excitation		$\frac{\dot{\theta}}{T} = \frac{\omega}{8B_p(1+L)} \times \left[1 - \frac{i4}{\pi}\ln ka + \frac{i8L}{\pi(1-\nu)}\left(\frac{h}{\pi\alpha}\right)^2\right]$	$P_s = \frac{\omega\lvert T\rvert^2}{16B_p(1+L)}$	$P_s = \frac{4\lvert\dot{\theta}\rvert^2 B_p(1+L)}{\omega\left\{1 + \left[\frac{4}{\pi}\ln ka - \frac{8L}{\pi(1-\nu)}\left(\frac{h}{\pi a}\right)^2\right]^2\right\}}$

Table 2. Continued

System	Onset of infinite behaviour	Largest point mobility of finite system	Ratio of finite system maximum to infinite system	Wavenumber (k)	Displacement of structure
Beam longitudinal wave motion; force excitation	$\omega > \frac{\pi}{\eta l}\sqrt{\left(\frac{E}{\rho}\right)}$	$\beta_l = \frac{2}{\pi A \eta \sqrt{E\rho}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{4}{\pi\eta}$	$k = \omega\sqrt{\left(\frac{\rho}{E}\right)}$	$\xi(y) = \frac{-\mathrm{i}F\,\mathrm{e}^{-iky}}{2\omega A\sqrt{E\rho}}$
Beam torsional wave motion; torque excitation	$\omega > \frac{\pi}{\eta l}\sqrt{\left(\frac{GQ}{J}\right)}$	$\beta_l = \frac{2}{\pi\eta\sqrt{GQJ}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{4}{\pi\eta}$	$k = \omega\sqrt{\left(\frac{J}{GQ}\right)}$	$\theta(y) = \frac{-\mathrm{i}T\,\mathrm{e}^{-iky}}{2\omega\sqrt{GQJ}}$
Beam flexural wave motion; force excitation	$\sqrt{\omega} > \frac{4\pi}{\eta l}\left(\frac{EI}{\rho A}\right)^{1/4}$	$\beta_l = \frac{2l}{\pi^2\eta\sqrt{\rho AEI}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{4\sqrt{2}}{\pi\eta}$	$k = \sqrt{\omega}\left(\frac{\rho A}{EI}\right)^{1/4}$	$\xi(y) = \frac{-\mathrm{i}F}{4EIk^3}[\mathrm{e}^{-iky} - \mathrm{i}\,\mathrm{e}^{-ky}]$
Beam flexural wave motion; torque excitation	$\sqrt{\omega} > \frac{4\pi}{\eta l}\left(\frac{EI}{\rho A}\right)^{1/4}$	$\beta_l = \frac{2}{l\eta\sqrt{\rho AEI}}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{2\sqrt{2}}{\pi\eta}$	$k = \sqrt{\omega}\left(\frac{\rho A}{EI}\right)^{1/4}$	$\xi(y) = \frac{T}{4EIk^2}[\mathrm{e}^{-iky} - \mathrm{e}^{-ky}]$
Plate flexural wave motion; force excitation	$\omega > \frac{8}{\eta l_1 l_2}\sqrt{\left(\frac{B_p}{\rho h}\right)}$	$\beta_l = \frac{4l_1 l_2}{\pi^2\eta\sqrt{\rho h B_p}(l_1^2+l_2^2)}$	$\frac{\lvert\beta_l\rvert}{\lvert\beta_\infty\rvert} = \frac{32 l_1 l_2}{\pi^2\eta(l_1^2+l_2^2)}$	$k = \sqrt{\omega}\left(\frac{\rho h}{B_p}\right)^{1/4}$	Valid in far field only $\xi(r,\phi) = \frac{-\mathrm{i}F}{8B_p k^2}\sqrt{\left(\frac{2}{rk\pi}\right)}\,\mathrm{e}^{-\mathrm{i}(rk-\pi/4)}$
Plate flexural wave motion; torque excitation	$\omega > \frac{8}{\eta l_1 l_2}\sqrt{\left(\frac{B_p}{\rho h}\right)}$	$\beta_l = \frac{16 l_2}{\eta\sqrt{\rho h B_p}\,l_1(2l_2^2+l_1^2)}$ *		$k = \sqrt{\omega}\left(\frac{\rho h}{B_p}\right)^{1/4}$	Valid in far field only $(r,\phi) = \frac{T}{8B_p k}\sqrt{\left(\frac{2}{rk\pi}\right)}\,\mathrm{e}^{-(rk-\pi/4)}\sin\phi$

Table 2. Continued. List of symbols [23].

A	cross-sectional area of beam
B_p	bending stiffness of plate $[= Eh^3/12(1-\nu)]$
E	Young's modulus
F	force or pressure
GQ	torsional stiffness
I	second moment of area of beam
J	polar moment of inertia per unit length
L	parameter, from reference [9], which tends to unity for large a/h
P_a	power flow at station a
P_m	power flow associated with bending
P_s	power supplied by source
P_u	power flow associated with shear
T	torque
a	radius of disc over which torque applied to plate acts
e	2·718 ...
h	plate thickness
i	imaginary operator $(\sqrt{-1})$
i	subscript, instantaneous value
k	wavenumber
l	length of beam or plate; subscript, longitudinal waves
m	bending moment
r, ϕ	polar co-ordinates
t	time; subscript, torsional waves
u	shear force
v	velocity
x, y, z	Cartesian co-ordinates
β	mobility
δ	dirac delta function
η	loss factor
θ	angular displacement
ν	Poisson's ratio
ξ	displacement
ζ	imaginary component of flexural wavenumber $(1+i\zeta)$
π	3·1415 ...
ρ	density
λ	wavelength
ψ	mode shape
ω	radian frequency

4.9 Frequency response functions related to mobility

Other frequency-response functions, structural response ratios, which are used instead of mobility, are shown in Table 3.

Table 3. Equivalent definitions to be used for various kinds of measured frequency response functions related to mechanical mobility [19].

	Motion expressed as velocity	Motion expressed as acceleration	Motion expressed as displacement
Term	Mobility	Accelerance	Dynamic compliance
Symbol	$Y_{ij} = v_i / F_j$	a_i / F_j	x_i / F_j
Unit	$\mathrm{m/(N \cdot s)}$	$\mathrm{m/(N \cdot s^2) = kg^{-1}}$	m/N
Boundary conditions	$F_k = 0 ; k \neq j$	$F_k = 0 ; k \neq j$	$F_k = 0 ; k \neq j$
Comment	Boundary conditions are easy to achieve experimentally		
Term	Blocked impedance	Blocked effective mass	Dynamic stiffness
Symbol	$Z_{ij} = F_i / v_j$	F_i / a_j	F_i / x_j
Unit	$\mathrm{(N \cdot s)/m}$	$\mathrm{(N \cdot s^2)/m = kg}$	N/m
Boundary conditions	$v_k = 0 ; k \neq j$	$a_k = 0 ; k \neq j$	$x_k = 0 ; k \neq j$
Comment	Boundary conditions are very difficult or impossible to achieve experimentally		
Term	Free impedance	Effective mass (free effective mass)	Free dynamic stiffness
Symbol	$F_j / v_i = 1 / Y_{ij}$	F_j / a_i	F_j / x_i
Unit	$\mathrm{(N \cdot s)/m}$	$\mathrm{(N \cdot s^2)/m = kg}$	N/m
Boundary conditions	$F_k = 0 ; k \neq j$	$F_k = 0 ; k \neq j$	$F_k = 0 ; k \neq j$
Comment	Boundary conditions are easy to achieve, but results shall be used with great caution in system modelling		

4.10 Boundary conditions of experimentation

In experimental determination of mechanical mobility, a dynamic exciting force is applied to the structure at one point at a time. Thus the force boundary conditions are [19]

$$F_k = 0\,; k \neq j\,, \tag{15}$$

where j is the point of excitation and k denotes all other points of interest. When the same force boundary conditions are valid, measurement of the velocity response at point i and the exciting force at j yields the ijth element of the mobility matrix [19]:

$$Y_{ij} = (v_i / F_j)_{F_k = 0;\ k \neq j}\,. \tag{16}$$

These force boundary conditions can easily be achieved in practise.

Instead the elements of the impedance matrix Z are [19]:

$$Z_{ij} = F_i / v_j\,, \tag{17}$$

where the boundary conditions

$$v_k = 0\,; k \neq j \tag{18}$$

are very difficult or impossible to fulfil in practice. Eqs. (17) and (18) describe mathematically the definition of blocked impedance. These boundary conditions imply that it is not generally possible to determine experimentally the impedance matrix. The difference between force and velocity boundary conditions (Eqs. (15) and (18)) must be kept in mind when using mobility and impedance data.

4.11 Mechanical mobility and impedance matrices

4.11.1 Definitions

It is assumed that linear, elastic structures are being considered, so that superposition and normal calculation rules are valid. The set of mobility elements y_{ij} is defined as follow [24]:

$$v_i = \sum_j y_{ij} f_j\,. \tag{19}$$

The set of impedance elements is defined as follow [24]:

$$f_i = \sum_j z_{ij} v_j\,. \tag{20}$$

4.11.2 Mechanical mobility matrix

Mechanical mobility is a tensor (or tensor component) which describes the effects upon the resultant velocity of the application of a force or forces on a structure [24]. It can be presented in the frequency domain by a matrix Eq. [24]:

$$\mathbf{V}(\omega) = \mathbf{Y}(\omega)\mathbf{F}(\omega), \tag{21}$$

where $\omega = 2\pi f$ is the angular frequency, f is frequency, $\mathbf{F}(\omega)$ is the column vector of exciting forces at various points, $\mathbf{V}(\omega)$ is the column vector of velocity responses at the points of interest, and $\mathbf{Y}(\omega)$ is symmetric tensor of mobilities y_{ij}. This matrix Eq. (21) in the expanded form looks like

$$\begin{aligned} v_1 &= y_{11} f_1 + y_{12} f_2 + y_{13} f_3 + \cdots, \\ v_2 &= y_{21} f_1 + y_{22} f_2 + y_{23} f_3 + \cdots, \\ v_3 &= y_{31} f_1 + y_{32} f_2 + y_{33} f_3 + \cdots, \\ v_4 &= y_{41} f_1 + y_{42} f_2 + y_{43} f_3 + \cdots, \text{etc.} \end{aligned} \tag{22}$$

The term $y_{ij} f_j$ defines a velocity at point i caused by a force acting at a point j. If this velocity is noted by $\bar{v}_{ij}$, then [24]

$$v_i = \sum_j \bar{v}_{ij} \,. \tag{23}$$

It is seen from this equation that the mobility is a concept that sums velocity response [24].

The elements of the matrix **Y** can be measured by applying the forces one at a time to each point of interest allowing the structure to response as it chooses, and the individual elements are obtained as the complex ratio of the particular velocity response to the single exciting force. If for example only the force f_2 is applied, then Eq. (22) would reduce to the set [24]

$$\begin{aligned} \bar{v}_{12} &= y_{12} f_{2,} \\ \bar{v}_{22} &= y_{22} f_{2,} \\ \bar{v}_{32} &= y_{32} f_{2,} \end{aligned} \tag{24}$$

and so on, since $f_k = 0,\ k \neq 2$. Then the element y_{12} is the obtained as the complex ratio

$$y_{12} = \bar{v}_{12} / f_2 \text{, etc.} \tag{25}$$

The reciprocity theorems of vibrations hold, and thus $y_{ij} = y_{ji}$ [24].

4.11.3 Impedance matrix

Impedance is a tensor (or tensor component) which describes the effects upon the resultant force (or several forces) of the application of a velocity or velocities on the structure [24]. This can be represented by the matrix equation

$$\mathbf{F}(\omega) = \mathbf{Z}(\omega)\mathbf{V}(\omega), \tag{26}$$

where $\omega = 2\pi f$ is the angular frequency, f is frequency, $\mathbf{F}(\omega)$ is the column vector of resultant forces f_i, $\mathbf{V}(\omega)$ is the column vector of applied velocities v_j, and $\mathbf{Z}(\omega)$ is symmetric tensor of impedances z_{ij}. This matrix equation can be expanded as follow [24]

$$\begin{aligned} f_1 &= z_{11}v_1 + z_{12}v_2 + z_{13}v_3 + \cdots, \\ f_2 &= z_{21}v_1 + z_{22}v_2 + z_{23}v_3 + \cdots, \\ f_3 &= z_{31}v_1 + z_{32}v_2 + z_{33}v_3 + \cdots, \\ f_4 &= z_{41}v_1 + z_{42}v_2 + z_{43}v_3 + \cdots, \text{etc.} \end{aligned} \tag{27}$$

The term $z_{ij}v_j$ defines a force at the point i caused by an applied velocity at the point j. If this force is called $\bar{f}_{ij}$, then

$$f_i = \sum_j \bar{f}_{ij}. \tag{28}$$

From Eq. (28) it is seen, that impedance is a concept that sums force response [24].
When determining the elements of impedance matrix $\mathbf{Z}$ the velocities are applied one at a time to each point of interest, the structure is not allowed to response freely, instead it is constrained to have zero velocity at the points where other velocities will be applied, and the individual elements are obtained as the complex ratio of the particular force response to the single exiting velocity [24]. Consider an example were only the velocity v_2 is applied on the structure at the point 2 then the Eq. (27) will reduce to the set

$$\begin{aligned} \bar{f}_{12} &= z_{12}v_2, \\ \bar{f}_{22} &= z_{22}v_2, \\ \bar{f}_{32} &= z_{32}v_2, \end{aligned} \tag{29}$$

and so on, since $v_k = 0,\ k \neq 2$. In this case $\bar{f}_{j2}$ $(j \neq 2)$ is the blocking (constraining) force at the point j, when the structure is excited by a velocity at the point 2, which is necessary to constrain the velocity at the point j to zero, and $\bar{f}_{22}$ is the force which results from the excitation motion at the point 2 [24]. The element of the impedance matrix is obtained from

$$z_{j2} = \bar{f}_{j2} / v_2. \tag{30}$$

According to reciprocity $z_{ij} = z_{ji}$ [24]. From Eqs. (21) and (26) it is easily seen that $\mathbf{Z} = \mathbf{Y}^{-1}$. According to matrix calculation the individual elements of impedance matrix are not the arithmetic reciprocals of the elements of the mobility matrix, and vice versa, that is $z_{ik} \neq y_{ik}^{-1}$ except in the trivial case of only one point [24].

Notice that the point means the location and the corresponding direction. If in a system the number of points is *N,* then the order of vectors is *N* and the order of matrices is *N* x *N*. The concept of immittance (impedance or admittance) and transmission matrices in the context of the vibration of mechanical systems is discussed in [26]. The mobilities on the contrary to the impedances of a given structure do not interdepend upon both the location and number of points of interest [24]. Mobilities describe invariant characteristics of the whole structure; instead impedances describe only substructures. During the mobility measurements the observations made anywhere on the system do not affect each other. Thus the mobility element y_{ij} remains the same although measurements are made at other points. Instead the impedance elements depend upon the number of observation points and the set of blocking forces used. So the impedance elements cannot be considered as invariant characteristics of the structure [24].

In some applications a complete mobility matrix has to be measured for the description of the dynamic characteristics of a structure. So translational forces and motions along three mutually perpendicular axes as well as moments and rotational motions may be required to be measured depending on the applications. These measurements result in a 6 x 6 mobility matrix for each measurement location. For *N* measurement locations this means a full 6*N* x 6*N* mobility matrix.

However, in practice only seldom the full mobility matrix needs to be measured. Usually it suffices to measure only the driving-point mobility in the excitation location and a few transfer mobilities in locations of interest on the structure. Sometimes the dynamics of the system needs to be determined only in one co-ordinate direction, e.g., in vertical direction. Also in many practical engineering applications the influence of rotational motions and moments is negligible.

4.12 Impedance and mobility of a system of elements

The mechanical impedance or mobility of a system of connected elements at a point of interest can be calculated using Eqs. (4) through (7) and the properties of ideal mechanical elements presented in Table 4. Generally this results in a complex impedance or mobility function which represents an equivalent system for the original system of connected elements. This equivalent system has same dynamic characteristics as the original system. According to Eq. (5) complex impedance represents parallel-connected elements from which the real part represents a purely resistive element and the imaginary part a purely reactive element. A complex mobility represents according to Eq. (6) series connected elements from which the real part represents a purely resistive element and the imaginary part a purely reactive element. These two elements of an equivalent system need not to be the ideal resistance, spring or mass. For instance the resistance may vary with frequency and the reactance may behave springlike, masslike or even be zero depending on frequency. The characteristics of real physical elements may be non-linear and all elements have some mass, and at very high frequencies when the wavelength becomes comparable with the element dimensions the wave phenomena may arise [28].

Table 4. Impedance and mobility of ideal lumped elements, parallel and series connected elements.

Ideal mechanical element or system	Impedance [(N·s)/m]	Mobility [m/(N·s)]
mass	$i\omega m$	$\frac{1}{i\omega m} = \frac{-i}{\omega m}$
resistance	c	$1/c$
spring	$\frac{k}{i\omega} = \frac{-ik}{\omega}$	$\frac{i\omega}{k}$
parallel connected elements	$Z_\mathrm{p} = \sum_{i=1}^{n} Z_i$	$\frac{1}{Y_\mathrm{p}} = \sum_{i=1}^{n} \frac{1}{Y_i}$
series connected elements	$\frac{1}{Z_\mathrm{s}} = \sum_{i=1}^{n} \frac{1}{Z_i}$	$Y_\mathrm{s} = \sum_{i=1}^{n} Y_i$

5 Impedance and mobility parameters

5.1 Impedance parameters

In a generalized two-connection system shown in Figure 4 a force F_1 is applied to excite the input and as a result a velocity v_1 is obtained. The ratio F_1/v_1 is called the input impedance. If instead a force F_2 is applied to excite the output, then a velocity v_2 results. The ratio F_2/v_2 is called the output impedance. When the force F_1 is applied to the input and the output velocity v_2 results, then the ratio F_1/v_2 is the reverse transfer impedance. When the force F_2 is applied to excite the output and the velocity v_1 results, then the ratio F_2/v_1 is called the forward transfer impedance. These definitions can be

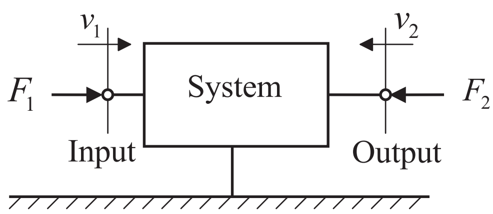


Figure 4. Generalized two-connection system [28].

applied to get the impedance parameters [28]:

$$Z_{11} = \left.\frac{F_1}{v_1}\right|_{v_2=0}, \qquad (31)$$

where Z_{11} is the input impedance under the condition $v_2 = 0$ meaning that the output is clamped (or blocked), i.e., connected to a rigid point. Similarly one gets the output impedance

$$Z_{22} = \left.\frac{F_2}{v_2}\right|_{v_1=0}. \qquad (32)$$

The reverse transfer impedance Z_{12} is

$$Z_{12} = \left.\frac{F_1}{v_2}\right|_{v_1=0}, \qquad (33)$$

where the input is clamped and F_1 is the force required to keep the input velocity $v_1 = 0$. The forward transfer impedance Z_{21} is

$$Z_{21} = \left.\frac{F_2}{v_1}\right|_{v_2=0}, \qquad (34)$$

where the output is clamped and F_2 is the force required to keep the output velocity $v_2 = 0$.

A two connection passive system can be represented by a black box, which is attached to an inertial reference (see Figure 5). If the elements in this system are linear and bilateral, then the relationship between the forces and velocities at the connection points

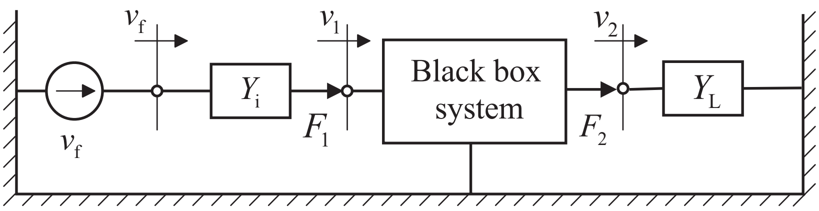


Figure 5. A mechanical system described as a black box, which has two connections and which is attached to an inertial reference. The vibration source is presented by a Norton equivalent system having free velocity v_f and internal mobility Y_i. The load is presented by its mobility Y_L. The forces and velocities at connection points 1 and 2 are noted by F_1, v_1 and F_2, v_2, respectively [27].

1 and 2 can be expressed as [28]:

$$F_1 = Z_{11} v_1 + Z_{12} v_2, \qquad (35)$$

$$F_2 = Z_{21} v_1 + Z_{22} v_2, \qquad (36)$$

where the Z_{ii}'s are the impedance parameters. When the forces and the impedance parameters are known, the velocities can be obtained by solving Eqs. (35) and (36).

5.2 Mobility parameters

The mobilities of a generalized system shown in Figure 4 are [28]:
The input mobility Y_{11} is

$$Y_{11} = \left.\frac{v_1}{F_1}\right|_{F_2=0}, \tag{37}$$

where the output is free without no restraining force, i.e., $F_2 = 0$.

The output mobility Y_{22} is

$$Y_{22} = \left.\frac{v_2}{F_2}\right|_{F_1=0}, \tag{38}$$

where the input is free without no restraining force, i.e., $F_1 = 0$.

The reverse transfer mobility Y_{12} is

$$Y_{12} = \left.\frac{v_1}{F_2}\right|_{F_1=0}, \tag{39}$$

where v_1 is the velocity of the free input, $F_1 = 0$, when the force F_2 is applied to excite the output.

The forward transfer mobility Y_{21} is

$$Y_{21} = \left.\frac{v_2}{F_1}\right|_{F_2=0}, \tag{40}$$

where v_2 is the velocity of free output, $F_2 = 0$, when the force F_1 is applied to excite the input.

The relationships between the velocities and forces of the black box system shown in Figure 5 can also be expressed using the mobility parameters Y_{ii} as [28]:

$$v_1 = Y_{11}F_1 + Y_{12}F_2, \tag{41}$$

and

$$v_2 = Y_{21}F_1 + Y_{22}F_2. \tag{42}$$

When the velocities at the connection points and the mobility parameters are known, then the forces are obtained by solving the Eqs (41) and (42).

In the mechanical black box system presented in Figure 5 the vibration source is described by Norton equivalent with free velocity v_f and internal mobility Y_i and the load is given by its load mobility Y_L. The internal mobility of the vibration source and the mobility of the load can be included with the black box mobilities by measuring or calculating the mobility parameters with the load mobility Y_L and internal mobility Y_i in place. In doing so the Eqs. (41) and (42) become [28]

$$v_\mathrm{f} = Y'_{11}F_1 + Y_{12}F_2, \tag{43}$$

and

$$0 = Y_{21}F_1 + Y'_{22}F_2. \tag{44}$$

Now the forces and velocities are considered at points 1′ and 2. The velocity at point 1′ v_1 becomes v_f and the velocity $v_2 = 0$ because no external force is applied. The mobility Y'_{11} is Y_{11} obtained from Eq. (37), and Y'_{22} is Y_{22} obtained from Eq. (38) with Y_L in place [28]. The transfer mobilities are not effected by the external connections. From Eqs. (43) and (44) one obtains the forces [28]

$$F_1 = \frac{v_\mathrm{f}Y'_{22}}{Y'_{11}Y'_{22} - Y_{12}Y_{21}} \tag{45}$$

and

$$F_2 = \frac{-v_\mathrm{f}Y_{21}}{Y'_{11}Y'_{22} - Y_{12}Y_{21}}. \tag{46}$$

The force $F_\mathrm{L} = F_2$ is applied to the load giving rise to load velocity $v_\mathrm{L} = F_2Y_\mathrm{L}$. The force at the input connection point 1 is F_1, since the internal mobility Y_i of the vibration source transmits this force and the velocity at the point 1 is $v_1 = v_\mathrm{f} - F_1Y_1$.

6 Transfer matrices

Usually many structural elements are connected to each other in a built-up structure. Mechanical mobilities of geometrically simple structural elements are known through impedance or directly as mobility [21], [23]. However, many times one needs to know what are the mobility functions of a built-up system. Consider a simple case of connected beam elements shown in Figure 6. The forces and velocities at the end of a beam are seen in this figure. Note that the forces are defined positive when pointing towards the beam. Instead, the positive directions of velocities are in the same direction pointing right in the figure.

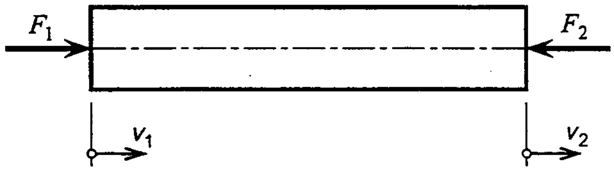


Figure 6. Forces acting on a beam [25].

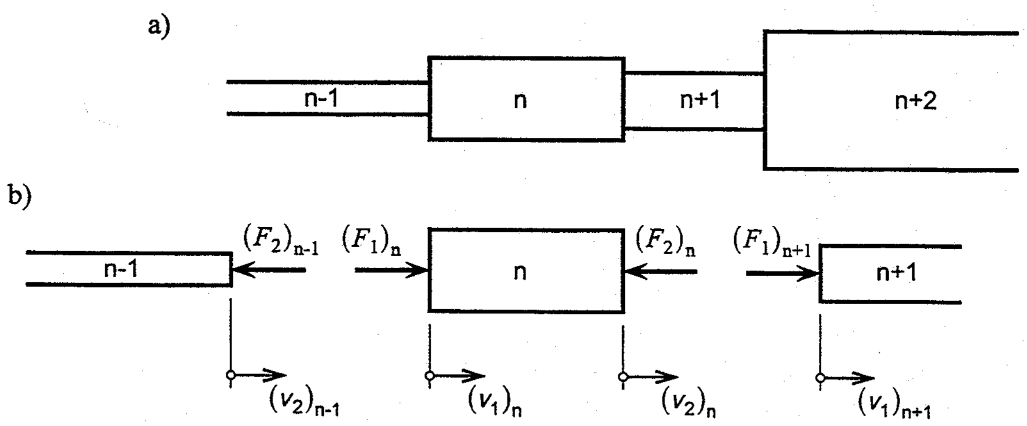


Figure 7. Forces and velocities at junctions between coupled beams [25].

The equations between the forces and velocities can be written with mobilities $Y_{ij} = v_j / F_i$ omitting the time dependence in the notations as [25]

$$v_1 = F_1 \cdot Y_{11} - F_2 \cdot Y_{21}; \quad v_2 = -F_2 \cdot Y_{22} + F_1 \cdot Y_{12}, \tag{47}$$

where F_2 has a minus sign as compared to previous notations in Eqs. (41) and (42) because the positive direction of F_2 has been redefined. The quantities v_2 and F_2 at the output end can be solved as functions of v_1 and F_1 at the input end as [25]

$$v_2 = a_{11} \cdot v_1 + a_{12} \cdot F_1; \quad F_2 = a_{21} \cdot v_1 + a_{22} \cdot F_1, \tag{48}$$

where

$$\begin{aligned} a_{11} &= \cos(k_1 L); & a_{12} &= -\frac{i\omega \sin(k_1 L)}{SEk_1} \\ a_{21} &= -\frac{iSk_1 \sin(k_1 L)}{\omega}; & a_{22} &= \cos(k_1 L) \end{aligned}, \tag{49}$$

where ω is the angular frequency, k is the stiffness, L the length, S the cross-sectional area, and E is the Young's modulus of the beam element. The parameters E and k are complex due to losses and defined as $E = E_0(1 + i\eta)$ and $k = k_0(1 - i\eta / 2)$ for the loss factor values $\eta << 1$ [25]. With the notations above the Eq. (48) can be written in matrix form as

$$\begin{Bmatrix} v_2 \\ F_2 \end{Bmatrix}_n = [\mathrm{A}]_n \cdot \begin{Bmatrix} v_1 \\ F_1 \end{Bmatrix}_n , \tag{50}$$

where the subscript n refers to beam n.

When a number of beams are connected in series as in Figure 7, then at the junction between elements n and $n + 1$ the following boundary condition must be satisfied [25]

$$\begin{Bmatrix} v_2 \\ F_2 \end{Bmatrix}_n = \begin{Bmatrix} v_1 \\ F_1 \end{Bmatrix}_{n+1} ,$$

where v_2 and F_2 are velocity and force at the output end on the right side of beam n and v_1 and F_1 are the respective quantities on the input end on the left side of beam $n + 1$ seen in Figure 7. With the aid of Eq. (50) one obtains

$$\begin{aligned} \begin{Bmatrix} v_2 \\ F_2 \end{Bmatrix}_{n+1} &= [\mathrm{A}]_{n+1} \cdot \begin{Bmatrix} v_1 \\ F_1 \end{Bmatrix}_{n+1} = \\ &= [\mathrm{A}]_{n+1} \cdot \begin{Bmatrix} v_2 \\ F_2 \end{Bmatrix}_n = [\mathrm{A}]_{n+1} \cdot [\mathrm{A}]_n \cdot \begin{Bmatrix} v_1 \\ F_1 \end{Bmatrix}_n . \end{aligned}$$

This can be repeated for series connected beam elements and then the following equation is obtained [25]

$$\begin{Bmatrix} v_2 \\ F_2 \end{Bmatrix}_m = [\mathrm{A}]_m \cdot [\mathrm{A}]_{m-1} \cdots [\mathrm{A}]_1 \begin{Bmatrix} v_1 \\ F_1 \end{Bmatrix}_1 = [\mathrm{A}]_{\mathrm{tot1}} \cdot \begin{Bmatrix} v_1 \\ F_1 \end{Bmatrix}_1 . \tag{51}$$

If two of the quantities v_1, F_1, v_2 or F_2 at the outer ends of the connected beams are known, then Eq. (51) shows how the other two quantities are obtained. Further discussion of applications of Eq. (51) and transfer matrices for bending of beams and for infinite periodic beams is presented in [25].

7 Multiport methods

Ulf Carlsson has investigated at MWL (The Marcus Wallenberg Laboratory for Sound and Vibration Research) in Sweden mechanical mobility as a tool for the analysis of vibrations in mechanical structures in his doctoral thesis [5]. Theory of multiport methods with applications and verification to structure-borne sound transmission is handled thoroughly in this thesis. Applications in the thesis include a railway bogie model, a centrifugal separator model and a ship scale-model. Here this short discussion is districted to the ship model.

There are many possibilities to divide a built-up mechanical structure into a network of cascade coupled substructures. One example of this is the *I*-coupled system in Figure 8. In this example the system is treated as a cascade of three cascade coupled cascades. It is possible to analyze the cascades 1 and 2 in isolation before they are combined to the cascade 3 [5].

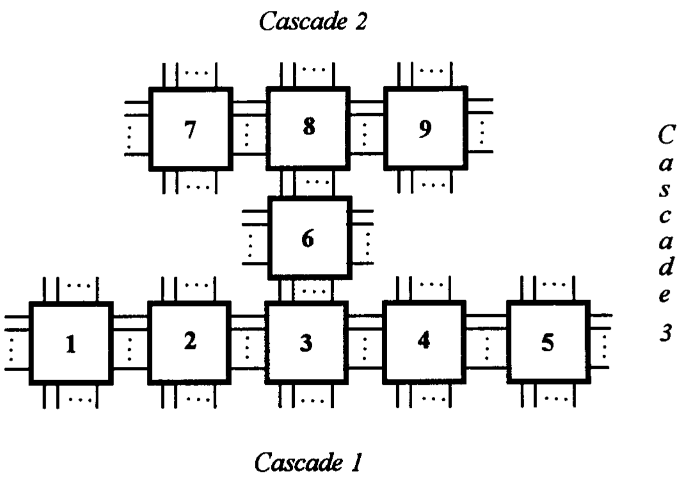


Figure 8. Example of a complex system, which is possible to treat as a sequence of cascade, coupled systems [5].

At MWL they have developed a software based on mobility techniques for the analysis of built-up structures. This program package was given name VIS (Vibrations In Structures) [5]. The package is able to analyze cascade coupled structures and it has many applications in research and development work [5]. The verification of the software has been made with parallel analyzes of known systems using independent calculation procedures. So far the software has not been commercially available.

Here is presented some results obtained using measurements and calculations using the software made for a ship model in scale 1:3 reported in [5, with reference to Masters thesis work of K. Haase 1993 TRITA-FKT Department of Vehicle Engineering, Royal Institute of Technology, Report TRITA-FKT 9309. Study of the mobility method to a structure-borne sound problem]. The hull of this model consisted of 1 mm and 3 mm aluminium plates welded together. The engine model was a steel frame filled with concrete. In the resilient mounting system of the engine model were used four vibration isolators type Novibra C-50/48 of stiffness type A [5]. The measurement points are presented in Figure 9 and Figure 10. From these points, seen in Figure 10, four points *E3*, *E4*, *E7* and *E8* were located on the stiff points on the intersections of stiffeners. *E1*, *E2*, *E5* and *E6* in Figure 10 denote the weak points located between stiffeners. The two receiver points denoted by r_1 and r_2 in Figure 9 were located on the deck structure.

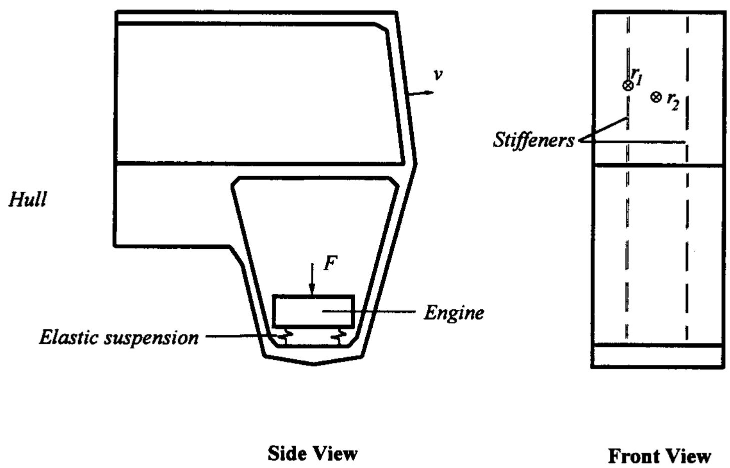


Figure 9. Model of the 1:3 scaled ship cross section with observation points right [5].

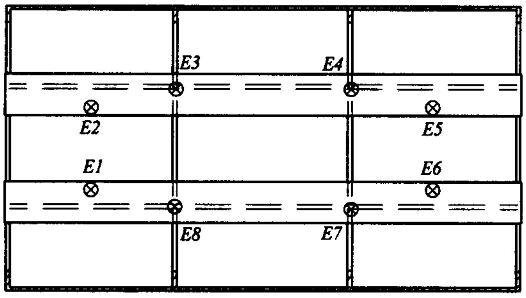


Figure 10. The engine foundation of the hull section seen from above [5].

Comparison of calculated and measured results is presented in Figure 11 to Figure 15. In many of the figures comparison is rather good between measured and calculated results in spite of errors included in the measured data. Also only one vibration isolator was measured and other isolator were assumed to have identical vibrational characteristics [5].

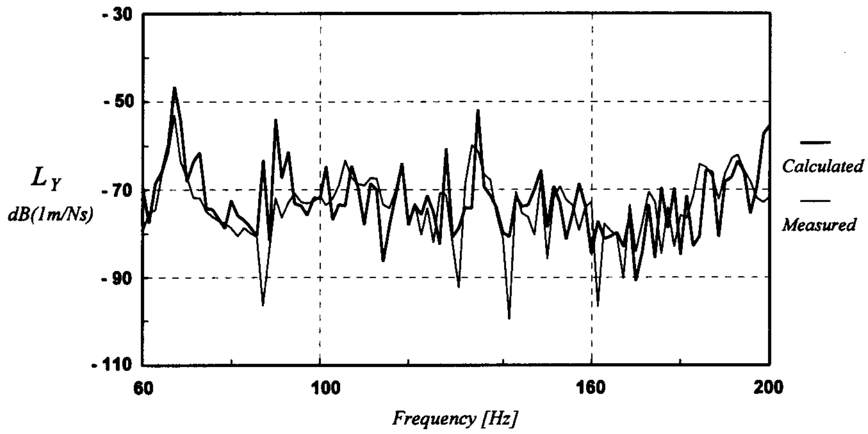


Figure 11. Engine without isolation system and engine mounted in stiff foundation points. Transfer mobility from excitation point on engine to stiff response point r_1 on hull in the low frequency range [5].

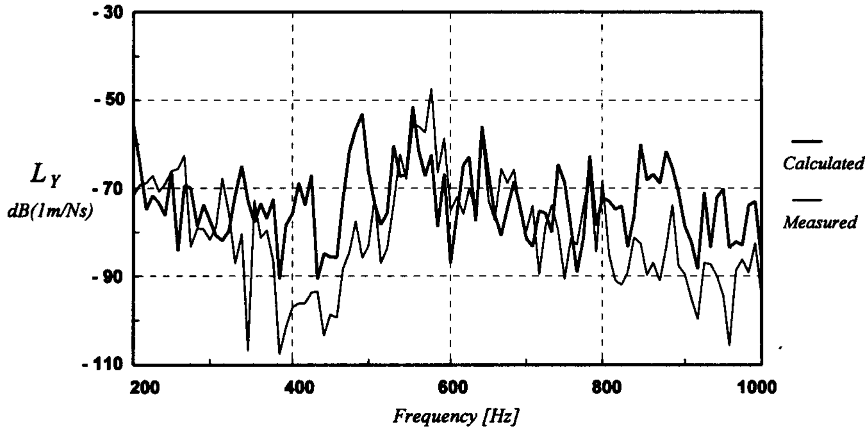


Figure 12. Engine without isolation system and engine mounted in stiff foundation points. Transfer mobility from excitation point on engine to stiff response point r_1 on hull in the intermediate frequency range [5].

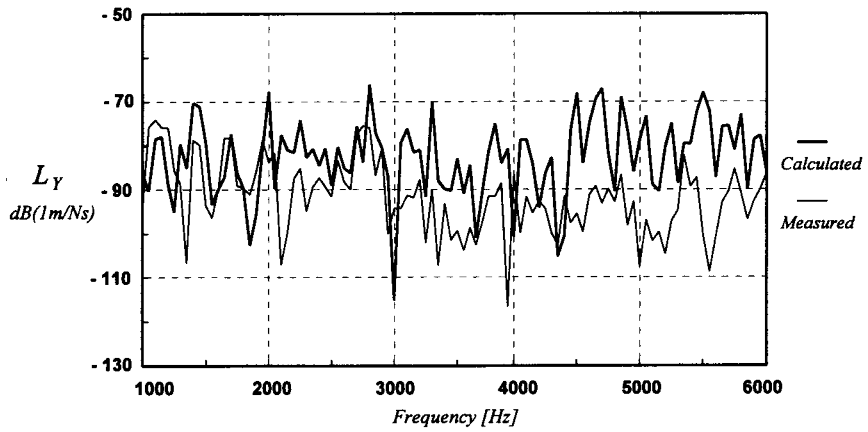


Figure 13. Engine without isolation system and engine mounted in stiff foundation points. Transfer mobility from excitation point on engine to stiff response point r_1 on hull in the high frequency range [5].

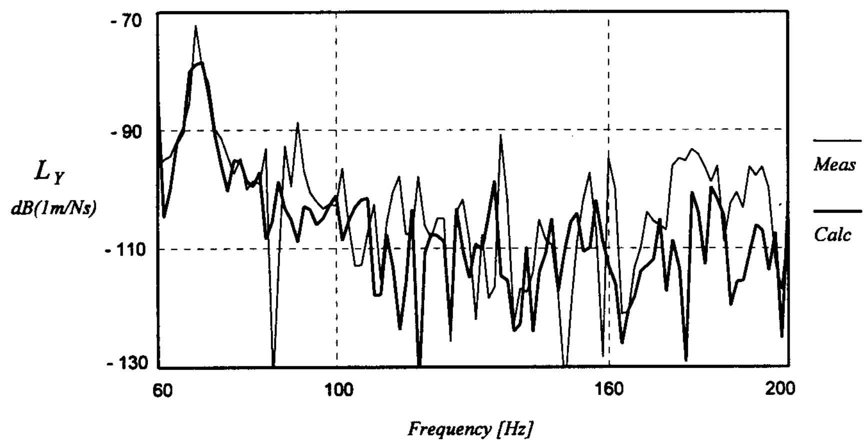


Figure 14. Engine with isolation system and engine mounted in weak foundation points. Transfer mobility from excitation point on engine to stiff response point r_1 on hull in the low frequency range [5].

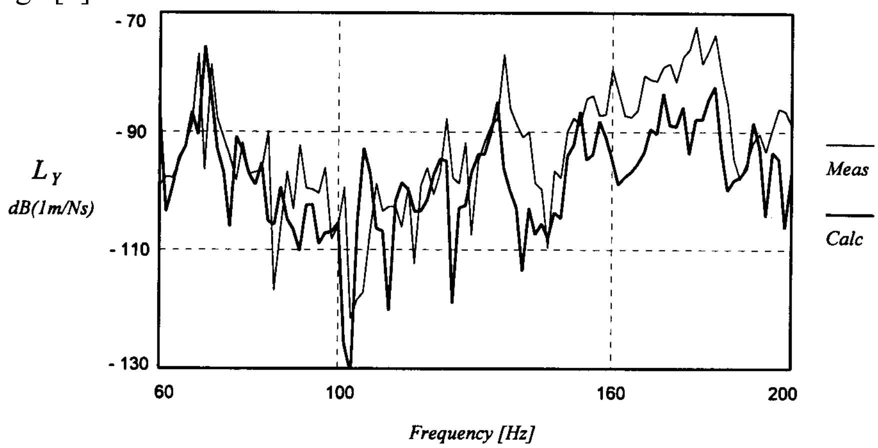


Figure 15. Engine with isolation system and engine mounted in weak foundation points. Transfer mobility from excitation point on engine to weak response point r_2 on hull in the low frequency range [5].

The comparizon of measured and calculated data obtained with the ship model and with other models as well shows that the mobility technique gives reliable results when input data are provided with sufficient accuracy. Mobility technique works rather well also when comparizon is made between resiliently and stiff mounted engines, but more reliable input data is needed than in the research work in [5] were available. At high frequencies it is difficult to obtain reliable input data due to the long transmission paths of structure-borne sound, to the small losses and to the high insertion loss of the resilient engine mounts [5].

8 Conclusions

This short literature review assures that the mobility technique can be used as a tool when analyzing vibrations in mechanical built-up structures. This is already an applicable tool in the low frequency range, but extension to higher frequencies requires more research work. Especially it could be applied when estimating the mobilities of seating structures. These mobility data is needed in pre-design of low noise machines and vehicles. However, the technique is promising and makes possible to utilize the substracturing technique with measured data of component characteristics. This requires that high quality data of the dynamic properties of different structural elements and that of resilient elements is available. Because the software package VIS (Vibrations In Structures) based on mobility technique is not yet commercially available. It is worth considering making research work aiming to develop for engineering applications similar robust software with a user-friendly interface.

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